Evidence of a $T = 0$ Quantum Critical Point Associated with the Crossover from Weak to Strong Localization

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A crossover between logarithmic and exponential temperature dependence of the conductance (weak and strong localization) has been observed in ultrathin films of metals deposited onto substrates held at liquid helium temperatures. The resistance at the crossover is well defined by the onset of a nearly linear dependence of conductance on thickness at fixed temperature in a sequence of *in situ* evaporated films. The results of a finite size scaling analysis treating thickness as a control parameter suggest the existence of a $T = 0$ quantum critical point which we suggest is a charge, or electron glass melting transition.

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Recently there has been considerable interest in electrical transport in two dimensional (2D) conductors. Since the work of the so-called "gang of four" it has been the conventional wisdom that in two dimensions weak disorder will localize electrons making it impossible for true metallic behavior to occur [1]. Although the classical work focused on the problem of noninteracting electrons, it has been widely accepted that its conclusions were applicable to interacting systems as well. This view has recently been strongly challenged by experimental studies of ultrahigh mobility low carrier density Si MOSFETs [2–5] which have revealed in an elegant way a metal-insulator transition (MIT) in a two dimensional system. Dobrosavljević *et al.* [6] in light of the work of Refs. [2–4] recently reconsidered the scaling theory of the MIT in two dimensional interacting systems and asserted that the existence of a 2D metal-insulator transition does not contradict any general principles. Another recent development in the physics of two dimensional conductors was the observation of Hsu *et al.* [7] of a change in the sign of the magnetoconductance of ultrathin films with strong spin-orbit interactions. This was correlated with a change in the temperature dependence of the conductance from logarithmic to exponential with decreasing temperature, *i. e.* the crossover between weak and strong localization, an effect which had been observed previously [8,9].

Here we present the result of a detailed study of the dependence of the conductance on thickness (d) and temperature in ultrathin Pd [8] and Bi films, metals with strong spin-orbit interactions. The success of a finite-size scaling analysis of these data leads us to suggest that there is a quantum critical point (QCP) in these films with the Drude conductance of the film as the control parameter. A QCP is found at zero temperature when a ground state is controlled by an external parameter [10]. It is our suggestion that the QCP we have found may be a charge or electron glass melting transition. This is motivated by the possibility that features of the description of vortex hopping in a two dimensional disordered superconductor may be similar to electron hopping in a two dimensional disordered conductor [11]. Although the vortex-glass phase transition in two dimensions is suppressed to $T = 0$, a 2D vortex glass correlation length which diverges in the limit of zero temperature has been reported [12]. This implies that there may be a QCP in this system as well. Finally there is ample evidence that high resistance films of metals whose conductance is governed by hopping exhibit glass-like properties, whereas low resistance films whose conductance is logarithmic in temperature, and which are not superconducting are not glass-like [13].

Our application of finite-size scaling to the weak-to-strong localization crossover was motivated by the observation that although the crossover appears gradual as a function of temperature as in Ref. [7], there is actually a very well-defined crossover conductance, G_{cr}, which is particularly sharp especially when the thickness, d, is the independent variable. We find $G_{cr} \sim (26 \pm 3k\Omega)^{-1}$ and $G_{cr} \sim (56 \pm 6k\Omega)^{-1}$ for Bi and Pd films respectively. Although G_{cr} is different in these materials, the critical exponents of the scaling analysis are nearly identical within experimental uncertainty.

Film thicknesses ranged from 8 to 20Å. Bismuth films were grown on a 9Å pre-deposited layer of amorphous Ge (a-Ge), whereas Pd films were grown directly on glazed ceramic substrates [8]. Depositions were carried out *in situ* under UHV conditions ($< 10^{-9}$ Torr) with substrate temperatures held below 15K. The ultra-high vacuum environment of the samples was sustained over an extended period so that film thickness could be built up by a series of sequential depositions without contamination. For a given set of films the deposition conditions (evaporation rate, source and substrate temperature, and background pressure) were fixed to avoid changes in morphology from film-to-film in a sequence. Resistance was measured between deposition increments using standard four-probe techniques with films current-biased at values less than 1nA. Current-voltage characteristics were linear up to currents at least ten times this value.

It is generally accepted that films of metals deposited on a-Ge substrates held at helium temperatures are con-

tinuous and homogeneous for two reasons: first, measurable conductance is often found at the order of monolayer coverages. Second, at very early stages of growth, the conductance becomes a linear function of thickness. Palladium films are known to wet glass substrates, also yielding films with measurable conductances at monolayer coverages. Close examination of our data indicates that a linear dependence of the conductance on thickness is never really found for films at the very earliest stages of their growth, even somewhat beyond monolayer coverage. An analysis of the onset of nearly linear behavior with increasing thickness is the subject of this work.

Figure 1 shows the conductance, G, of a Bi film as a function of d at several representative temperatures, and the derivative of $G(d)$ with respect to d at 3.3K. These data are obtained by sorting data on $G(T)$ obtained from films of different thicknesses, and considering them as a function of d at fixed temperature. The derivative is then computed numerically. Graphs of the derivative, of which only one example is shown, exhibit a pronounced change at a temperature dependent thickness d_c which always occurs at a constant conductance G_{cr}. Films with thicknesses greater than d_c exhibit a nearly linear dependence on thickness. The values of the critical conductance for Bi and Pd films, $G_{cr} \sim (26 \pm 3k\Omega)^{-1}$, and $(56 \pm 6k\Omega)^{-1}$, respectively, are very close to e^2/h and $e^2/2h$.

Explanations of G_{cr} being a consequence of a structural change occurring at a particular thickness can be ruled out. If such were the case, the boundary between the linear and non-linear dependences would occur at a fixed thickness independent of temperature rather than at a fixed conductance G_{cr}. It should also be mentioned that the conductivity near treshold is not described by a percolation model with the conductance given by a power law in $(d-d_p)$, where d_p is the thickness at the threshold for conductance. In this approach the average thickness is assumed to be proportional to the surface coverage [14]. It is actually given by an exponential in $1/d^{\alpha}$, although the power α is somewhat uncertain because of the limited range of the data. A value of $\alpha = 1/2$ would be consistent with variable range hopping [15].

Further support of the view that the crossover is an intrinsic effect comes from the temperature dependence of the conductance at fixed thickness, which is not shown. At low temperatures, and in the thinnest films at all temperatures, the conductance is activated. For resistances below $50 - 100k\Omega$ it is given by the well-known functional form of the soft Coulomb gap model $G \propto \exp[-(T_0/T)^{1/2}]$ [15]. Here T_0 is related to the localization length, L_{Loc}, through $k_B T_0 = e^2/\kappa L_{Loc}$, where κ is the dielectric constant. For resistances above 50-100kΩ the exponent is closer to 4/5 rather than 1/2 which may be evidence of collective variable range hopping of charges, or may be the result of a crossover from variable range to fixed range hopping with increasing resistance. At high temperatures for all but the most resistive films the conductance varies linearly with $\ln T$ consistent with the expectations of the electron interaction picture [16]. Because the crossover in temperature between logarithmic and activated conductance is not as sharp as the thickness-dependent crossover, it has not been used to determine G_{cr}. The fact that the crossover is seen at the same G_{cr} in both the temperature and thickness dependence of the conductance suggests that the effect is not some manifestation of percolation on an atomic scale [17] . We have also observed a change in the sign of the magnetoresistance in Bi films from negative to positive with increasing thickness which is qualitatively consistent with Ref. [7]. However, it does not occur at the same value of the resistance as the crossovers described above. This delicate issue is currently under study.

The above observations led us to investigate the possibility of a quantum critical point (QCP). In this scenario for $T > 0$ there would be a line of crossovers at $d = d_c(T)$, terminating in a critical point at $T = 0$. It is sufficient for a QCP that the finite temperature boundary be a crossover. It is not necessary that it be a line of first order transitions [18]. Because we can determine G_{cr} from $G(d)$ with precision we take thickness d as the control parameter, and define d_c as a temperature dependent critical thickness. The use of a temperature-dependent parameter will be seen to be a convenience, with the Drude conductance at high temperatures, which is temperature independent, being the actual control parameter.

The distance from the critical thickness, $\delta = d - d_c$ is then the relevant field. Standard arguments [10,18,19] suggest that the scaling form for a two dimensional system is

$$G(\delta, T) = G_{cr} F(\delta T^{-1/\nu z}) \tag{1}$$

Here $F(x)$ is a universal scaling function such that $F(0) = 1$, ν is the coherence length exponent, and z is the dynamical critical exponent. To proceed with the analysis we rewrite Eq. 1

$$G(\delta, t) = G_{cr} F(\delta t) \tag{2}$$

and define $t \equiv T^{-1/\nu z}$. The parameter $t(T)$ is treated as an unknown variable to be determined at each temperature to obtain the best collapse of all the data [19]. The exponent νz is then found from the temperature dependence of t, which must be a power law in temperature for the procedure to make physical sense. We should note that the present scaling procedure does not require detailed knowledge of the functional form of the temperature dependence of the conductivity, or prior knowledge of the critical exponents. It is simply based on the data which includes an independent determination of d_{cr} at each temperature.

The results of the analysis for both Bi and Pd are shown together in Fig. 2. The crossover thickness $d_c(T)$, determined from the data like that of Fig. 1, increases logarithmically with decreasing temperature, as shown on Fig. 3. The temperature dependence of the parameter t, shown on Fig. 4 is consistent with a power law, with $\nu z = 6.9 \pm 0.7$ and 7.2 ± 1.0 for Bi and Pd, respectively.

Although our analysis is carried out for convenience with an apparently temperature-dependent critical parameter, it can be cast in terms of a temperature independent one. Adding metal sequentially to a quench-condensed film controls disorder through its smoothing of the random potential. What probably also occurs is an increase in the carrier concentration, and thus an increase in the screening. These effects are measured directly by the conductance, and indirectly by the thickness. Although G is always a temperature dependent quantity its value at high temperatures is close to the Drude conductance G_D, which is proportional to the thickness and constant for a given film. A natural control parameter for the QCP would be G_D, with the relevant field being $\delta_D = |G_D - G_{cr}|$. Because all the films we used in the scaling analysis exhibit lnT behavior in their conductances, with $G \propto d$ at high temperatures, the control parameter used in our analysis, δ, is effectively the same as δ_D provided that $d_c(T)$ is a logarithmic function of temperature as is demonstrated in Fig.3. In other words, temperature dependence of d_c is not a consequence of an underlying finite temperature phase transition, it only reflects the fact that d_c is related to G_{cr} through a temperature dependent factor.

The crossover will always occur in films which are effectively two dimensional at nonzero temperature because the inelastic scattering length, l_{in} , will always be greater than the film thickness. One can crudely estimate l_{in} by equating it to the localization length at the crossover conductance [20]. This is tantamount to matching the length scales of weak and strong localization at the crossover. Taking the dielectric constant to be that of Ge, $\kappa = 16$, one finds $l_{in} \sim 100\mathring{A}$ in the thinnest films at the crossover. The inelastic scattering length, l_{in}, is a power law in temperature proportional to $T^{-1/2}$ which would increase from $\sim 100\mathring{A}$ with decreasing temperature. Since d_c increases as $\propto \ln T$ from $\sim 12\mathring{A}$ over the measured temperatures, one would expect d_c to remain smaller than the localization length at all nonzero temperatures. Examination of the data of Fig. 2 suggests that the temperature dependence of $d_c$may be saturating at the low end of the temperature range, which may mean that $d_c$does not increase indefinitely and is bounded at temperatures below the measured range.

In previous work done by our group [8] and others [7], the data for the thickness and temperature dependences of sequences of films have been scaled by adjusting only the temperature axis. This procedure did not include a specific recognition of the crossover phenomena shown in Fig. 1, and involved forcing a functional form on high temperature data which was slightly different from that of localization theory [1].

The QCP separates films whose conductances exhibit logarithmic from those with exponential dependence on temperature at nonzero temperatures. Although both types of films are nominally insulating, the analysis would suggest that they have either fundamentally different ground states or perhaps different dynamical behavior. The existence of a serious distinction between insulators exhibiting logarithmic and exponential dependence on temperature was first noted in the work of Burns and Chaikin [21] on thermopower in Pd and Pd-Au films. They found a sharp crossover in the thermopower from metallic-like to insulating-like behavior at a resistance near h/e^2. There is another important difference between films whose conductances exhibit a logarithmic dependence on temperature and those with an exponential dependence. The former appear *never* to be glass-like in their behavior, whereas the latter are unequivocally glass-like [13]. The evidence for this is the observation of long relaxation times and memory effects in a capacitive charging experiment. Glass-like behavior disappeared above a characteristic temperature roughly corresponding to the crossover from hopping to a logarithmic dependence of the conductance on temperature.

The magnitude of the critical exponent products we have found ($\upsilon z \sim 7$) seem large. In the absence of a detailed theory, it is hard to comment on this. However the value is not very different from the exponent products νz ranging between 4 and 10 found by Dekker *et al.* [12] in their study of the vortex-glass phase transition suppressed to zero temperature in a 2D superconductor. Thus, the large values of critical exponents in the present work would be plausible if charges in disordered ultrathin films and vortices in two dimensional superconductors behaved similarly. This line of argument leads us to *suggest* that the QCP inferred from the analysis we have presented may be the melting of a charge or an electron glass, with the higher-resistance phase being glass-like with activated transport, and the lower resistance phase being a liquid-like with logarithmic corrections to the classical conductivity from electron-electron interaction effects.

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FIG. 1. G vs. d for Bi films at 0.7K ($\circ$), 1.5K ($\diamond$), 3.3K ($\bullet$), 7.0K ($\triangle$) and 15.0K ($\bigtriangledown$). A fit to the linear part of the 3.3K curve is shown. Also plotted is the derivative of the 3.3K curve ($\times$). The horizontal dashed line represents the slope of the linear fit. A sharp bend in the derivative is observed very close to the thickness at which $G(d) = e^2/h$. Efforts to fit these curves to a percolation model, $G \propto (d - d_p)^\gamma$ were unsuccessful.

FIG. 2. Bi ($\circ$) and Pd ($\bullet$) data collapsed together. The poor collapse of the Pd data in the lower right is from the highest temperature curves ($T \geq 10K$), indicating that the scaling breaks down far from the $T = 0$ transition.

FIG. 3. d_c vs. T for Bi ($\circ$) and Pd ($\bullet$). d_c increases slowly (logarithmically) with decreasing temperature.

FIG. 4. T vs. $t(T)$ for Bi ($\circ$) and Pd ($\bullet$). The slopes of the power law fits yield $\nu z = 7.2 \pm 0.5$ and $\nu z = 6.9 \pm 0.5$ for Bi and Pd, respectively.

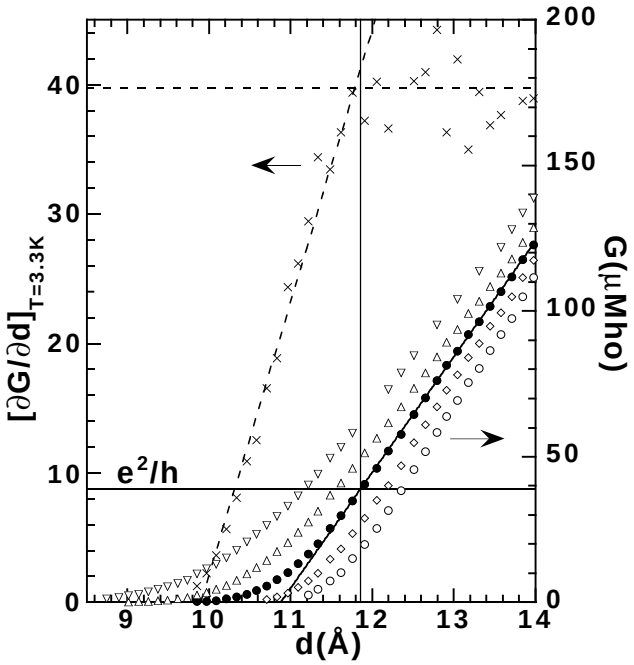
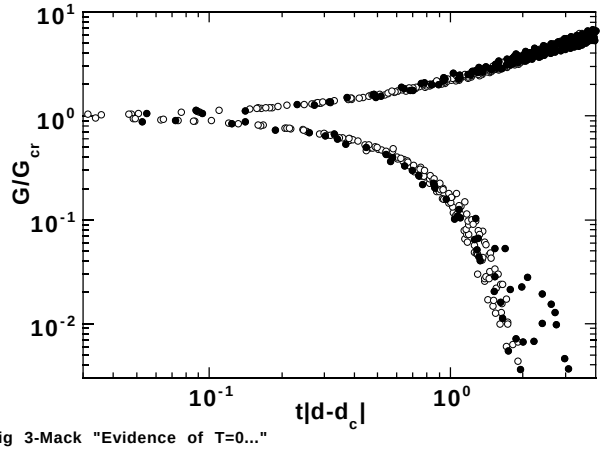


fig 1-Mack "Evidence of a T=0..."



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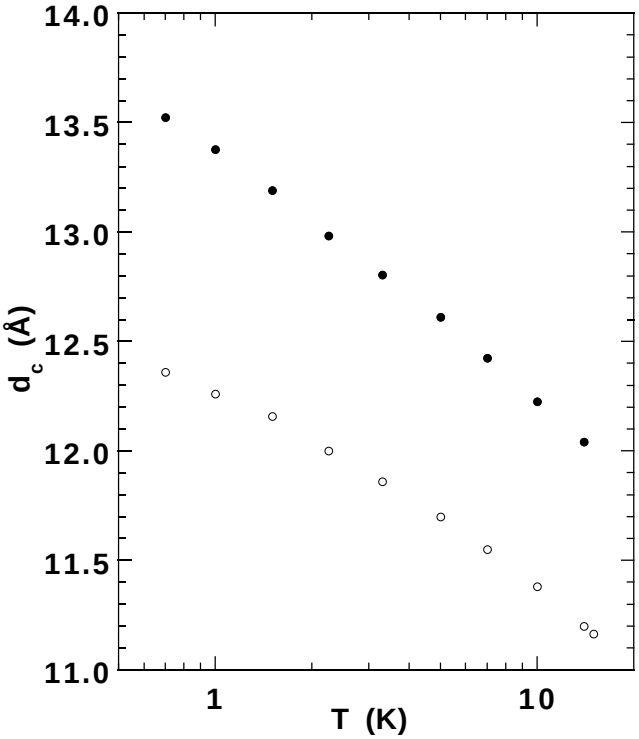

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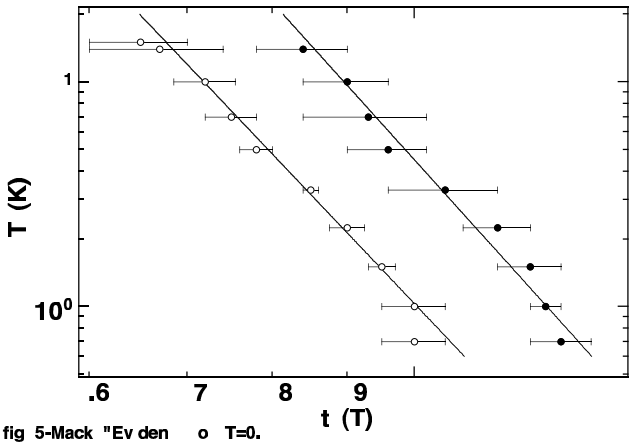

fig 5-Mack "Ev den o T=0.